transfer coefficient to fluid pumping power is not sufficient for seeking improved thermal performance. The $N_S$ criterion is a more adequate measure of thermodynamic imperfection and provides a more complete picture of how various design variables influence the thermal performance.

*Acknowledgements*—The author acknowledges the helpful comments received from Professors R. Greif and C. L. Tien of the University of California, Berkeley, and Professor W. M. Rohsenow of the Massachusetts Institute of Technology.

This work was supported by a post doctoral research fellowship awarded by the Miller Institute for Basic Research in Science, University of California, Berkeley.

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*Int. J. Heat Mass Transfer.* Vol. 21, pp. 658–660
0017-9310/78/0501-0658 $02.00/0

# A SIMPLE METHOD FOR TIME OF DRYING PREDICTION UNDER CONSTANT WET-BULB TEMPERATURE CONDITIONS

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(*Received* 11 *February* 1977 *and in revised form* 23 *September* 1977)

### NOMENCLATURE

| | |
|---|---|
| $A$, | drying surface [m$^2$]; |
| $B$, | dimensionless driving force as defined by equation (5); |
| $G_i$, | flow rate of a dry gas [kg/s]; |
| $k_Y$, | mass-transfer coefficient as defined by equation (2) [kg/m$^2$s]; |
| $k_Y^0$, | limiting mass-transfer coefficient at low humidity potentials [kg/m$^2$s]; |
| $Le$, | Lewis number [dimensionless]; |
| li$(u)$, | logarithmic integral; |
| $N_v$, | evaporation flux [kg/m$^2$s]; |
| $Pr^0$, | limiting Prandtl number for infinite dilution of vapour in a dry gas [dimensionless]; |
| $R_s$, | $= V_s/A$, specific volume of a solid [m]; |
| $S_c^0$, | limiting Schmidt number for infinite dilution of vapour in a dry gas [dimensionless]; |
| $T^*$, | wet-bulb temperature [K]; |
| $V_s$, | volume of a solid [m$^3$]; |
| $Y^*$, | saturated humidity mixing ratio at wet-bulb temperature [kg$_v$/kg$_i$]; |
| $Y$, | humidity mixing ratio in bulk gas [kg$_v$/kg$_i$]. |

Greek symbols

| | |
|---|---|
| $\alpha^0$, | convective heat-transfer coefficient [W/m$^2$K]; |
| $\beta$, | humidity level factor as defined by equation (6) [dimensionless]; |
| $\gamma$, | = 1.781072481, Euler's constant; |
| $\theta$, | time of drying [s]; |
| $\pi$, | humidity potential as defined by equation (7) [dimensionless]. |

Subscripts

| | |
|---|---|
| 1, | refers to entrance state; |
| 2, | refers to exit state; |
| $v$, | refers to water vapour; |
| $i$, | refers to dry gas. |

### PROBLEM FORMULATION

FOR TIME of drying prediction in convective dryers with parallel flow of gas and material under constant wet-bulb temperature conditions, the well-known expression: (see e.g. [1, p. 619])

$$\theta = \frac{G_i R_s}{V_s k_Y} \ln\left(\frac{Y^* - Y_1}{Y^* - Y_2}\right) \tag{1}$$

is most frequently used.

The relation (1) is derived from the simple macroscopic mass balance equation, while the drying rate is defined as:

$$N_v = k_Y(Y^* - Y) = k_Y \pi. \tag{2}$$

In deriving (1) it has been assumed that the mass-transfer coefficient $k_Y$ (2) does not depend on absolute humidity nor on humidity potential. Therefore, its value was treated as constant, under the defined flow conditions.

It can be emphasized that from the experiments (see e.g. [2]) it is quite evident that the coefficient $k_Y$ directly depends both on humidity potential and on humidity level. Therefore, by deriving equation (1) the mass-transfer coefficient $k_Y$ (2) cannot in general be considered to be constant.

Considering the fact that the water vapour evaporation is associated with a vapour diffusion process through an essentially still gas, the evaporation flux can be expressed by:

$$N_v = k_Y^0 \ln\left(\frac{1 + Y^*}{1 + Y}\right). \tag{3}$$

Equation (3) should be conceived as an expression defining the mass-transfer coefficient $k_Y^0$. We can recognize Stefan's, Colburn–Drew's [3] or Spalding's ([4], p. 193) diffusion model in it. In each of the mentioned hypotheses the mass-transfer coefficient $k_Y^0$ (3) does not depend on humidity potential nor on humidity level, but it is a function of flow conditions and surface geometry. The approach to drying rate analysis similar to (3), is accepted, and the conclusions connected with constancy of $k_Y^0$ are widely used in the book of Krischer ([5], pp. 248–254). As can be established by the analysis of newer experimental investigations on the evaporation of pure liquids under a wide range of the driving force (see e.g. [6]) the mass-transfer coefficient $k_Y^0$ is not quite constant. When the driving force changes the value of $k_Y^0$, however, changes less than that of $k_Y$.

The relation between the mass-transfer coefficients $k_Y$ and

$k_Y^0$ can be obtained by equalizing the equations (2) and (3):

$$k_Y = k_Y^0 \beta \ln\left(\frac{1+B}{B}\right). \tag{4}$$

The parameter similar to Spalding's generalized force [4] is denoted by $B$;

$$B = -\beta\pi, \tag{5}$$

while $\beta$ denotes the gas humidity level factor;

$$\beta = \frac{1}{1+Y^*}, \tag{6}$$

and $\pi$ represents the humidity potential;

$$\pi = Y^* - Y. \tag{7}$$

For the limiting case defined by conditions under which the parameter $|B|$ has a very small value, from equation (4) we get:

$$\lim_{|B|\to 0} k_Y = k_Y^0 \beta. \tag{8}$$

The basic idea of this work is to get an expression for drying time prediction under constant wet-bulb temperature conditions using the drying rate definition (3) instead of (2), assuming that the mass-transfer coefficient $k_Y^0$ is constant instead of $k_Y$.

By using (3), the macroscopic mass balance equation can be written as:

$$\frac{k_Y^0 V_s}{R_s} \ln\left(\frac{1+Y^*}{1+Y}\right) d\theta = G_i \, dY. \tag{9}$$

An expression for the time of drying is obtained by simple integration of equation (9) under the conditions:

$\theta = 0$: $Y = Y_1$ (entrance state),

$\theta = \theta^0$: $Y = Y_2$ (exit state),

$Y^*$ = constant (drying at constant wet-bulb temperature).

## TIME OF DRYING

Direct integration of (9) over the time interval $\theta^0$ while the humidity changes from $Y_1$ to $Y_2$ leads to:

$$\theta^0 = \frac{G_i R_s}{V_s k_Y^0 \beta}[\mathrm{li}(1+B_1) - \mathrm{li}(1+B_2)]. \tag{10}$$

while $\beta$ is defined by (5) ($\beta$ = constant), and the values of the parameters $B_1$ and $B_2$ are defined by:

$$B_1 = -(Y^* - Y_1),$$
$$B_2 = -(Y^* - Y_2).$$

The solution (10) obtained for determination of the drying time is expressed through the logarithmic integral li($u$). The logarithmic integral represents a well-known function, whose tabulated values can be found in the literature about special functions (see e.g. [7]).

Determination of the mass-transfer coefficient $k_Y^0$ is often done on the basis of an analogy between heat- and mass-transfer processes (see e.g. [4]) through

$$k_Y^0/\alpha^0 = \frac{1}{c_{pi}}(Pr^0/Sc^0)^n = \frac{1}{c_{pi}} Le^{-n}. \tag{11}$$

The value of $n$ is between $\frac{1}{2}$ and $\frac{3}{4}$. The commonly accepted value for the system water–air with turbulent flow is $n = \frac{2}{3}$.

Using (11), equation (10) can be rewritten as:

$$\theta^0 = \frac{G_i R_s c_{pi}}{V_s \alpha^0 \beta} Le^n[\mathrm{li}(1+B_1) - \mathrm{li}(1+B_2)]. \tag{12}$$

It is obvious that the difference between the solutions (1) and (10) can be estimated by comparing the logarithmic integral li($1+B$) and the logarithmic ln($B$) functions as well as the mass-transfer coefficients $k_Y$ and $k_Y^0$.

The function li($1+B$) can be presented by the power series expansion:

$$\mathrm{li}(1+B) = \ln[-\gamma \ln(1+B)] + \sum_{k=1}^{\infty} \frac{[\ln(1+B)]^k}{k!k} \tag{13}$$

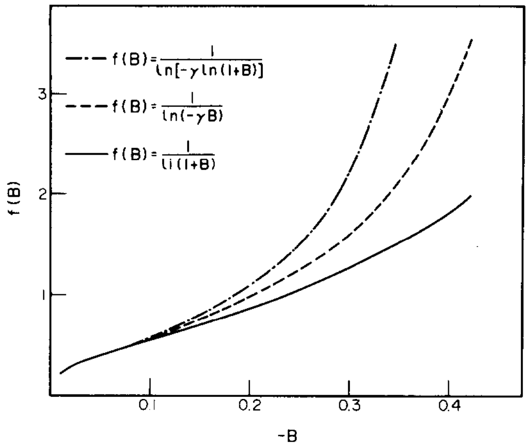

FIG. 1. Comparison between logarithmic integral (13) and its logarithmic approximations (14) and (15).

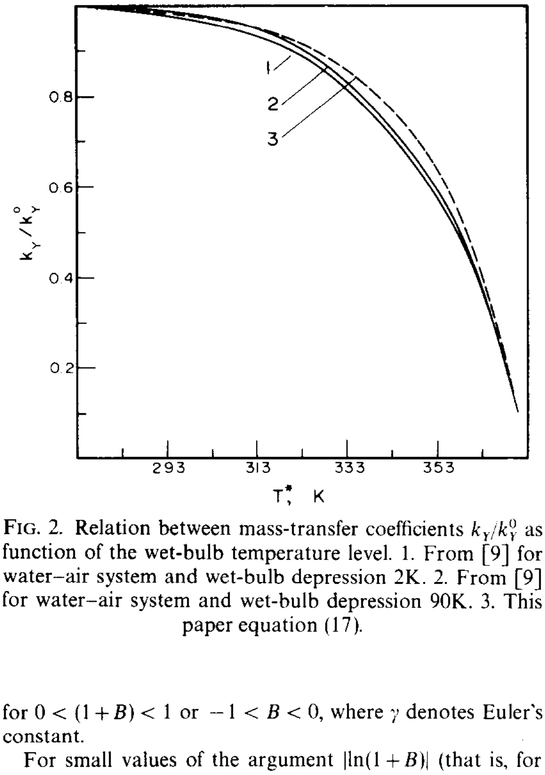

FIG. 2. Relation between mass-transfer coefficients $k_Y/k_Y^0$ as function of the wet-bulb temperature level. 1. From [9] for water–air system and wet-bulb depression 2K. 2. From [9] for water–air system and wet-bulb depression 90K. 3. This paper equation (17).

for $0 < (1+B) < 1$ or $-1 < B < 0$, where $\gamma$ denotes Euler's constant.

For small values of the argument $|\ln(1+B)|$ (that is, for small values of the potential $|-B|$) one has approximately:

$$\mathrm{li}(1+B) \cong \ln[-\gamma \ln(1+B)]. \tag{14}$$

When $|B|$ tends to zero ($|B| \to 0$) one has:

$$\mathrm{li}(1+B) = \ln(-\gamma B) \quad \text{for} \quad |B| \to 0. \tag{15}$$

The relation between the three above-mentioned functions (13)–(15) is graphically shown in Fig. 1.

From Fig. 1 it can be seen that (15) represents a better approximation of the logarithmic integral li($1+B$) than that of (14). Therefore, for small values of the parameter $|B|$, the drying time can be predicted, instead of using equation (12), from the approximate relation:

$$\theta^0 = \frac{G_i R_s}{V_s k_Y^0 \beta} \ln(B_1/B_2) = \frac{G_i R_s}{V_s k_Y^0 \beta} \ln\left(\frac{Y^* - Y_1}{Y^* - Y_2}\right). \tag{16}$$

## DISCUSSION

From the practical viewpoint of application of the obtained solutions it would be of interest to predict the drying regimes under which we should use the expressions (10) or (16) instead of (1).

We shall begin with the analysis of the relation between the solutions (10) and (16).

For illustration let us calculate the characteristic percentage differences which arise by approximation of (13) by means of (15):

for $B = -0.1$ one finds a difference of 2.7%,

for $B = -0.2$ one finds a difference of 9.9%.

In many operating regimes of convective dryers $|-B| < 0.2$. In such conditions approximation of the solution (10) can be allowed by means of equation (16).

Comparing equations (1) and (16) we can conclude that

$$\theta = \theta^0,$$

if we accept

$$k_Y = k_Y^0 \beta. \tag{17}$$

On the basis of (17) we can conclude that in the region mentioned (that is the regime wherein most of the commercial convective dryers are operating) the mass-transfer coefficient $k_Y$ can be considered to be independent of the humidity potential, but it is strongly dependent on saturation humidity level (or wet-bulb temperature). This conclusion has been fully confirmed by results of the comparison of experimental data and some theoretical solutions performed by Bobe *et al.* [8]. The conclusion quoted is in accordance with the theoretical analysis of Ashworth and Keey [9]. It is interesting to note that the dependence expressed by a very simple relation (17) is quantitatively in good accordance with their numerical results. This comparison is shown in Fig. 2.

As we have seen, the increase of absolute values of the parameter $|B|$ causes continuous enlargement of the difference between the solutions (10) and (16). For illustration, let us give some values of percentage difference between (10) and (16) for the parameter values $|B| > 0.2$:

for $B = -0.25$ one finds a difference of 17.2%,

for $B = -0.3$ one finds a difference of 23.6% etc.

From this analysis we can conclude that for the operating regimes with the parameter values $|B| > 0.2$ the solution (10) or (12) should be used for determination of the drying time.

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*Int. J. Heat Mass Transfer.* Vol. 21, pp. 660–662
0017-9310/78/0501-0660 $02.00/0

# PHOTOSENSITIZED REACTIONS IN ABSORBING–SCATTERING MEDIA WITHIN A PLANE SLAB*

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(*Received* 12 *July* 1977)

### NOMENCLATURE

| | |
|---|---|
| $\dot{a}'''$, | dimensionless local rate of energy absorption; |
| $c$, | scattering albedo; |
| $\dot{e}'''$, | dimensional local rate of energy absorption; |
| $\dot{\mathcal{E}}''$, | power absorbed within the slab per unit interfacial area; |
| $E_1$, | first order exponential integral; |
| $I$, | whole radiant energy incident in $x$; |
| $I_0$, | intensity of the incident radiation; |
| $m$, | $= (\beta+\sigma)s$ optical thickness; |
| $s$, | width of the slab; |
| $x$, | coordinate. |

Greek symbols

| | |
|---|---|
| $\beta$, | absorption coefficient; |
| $\theta$, | angle between a generical direction and the $x$-axis; |
| $\mu$, | $= \cos\theta$; |
| $\sigma$, | scattering coefficient. |

ATTENTION has been recently paid to the role of scattering in photochemical reactions [1–3].

Coupled absorbing and scattering phenomena are difficult to be handled since the radiant energy balance equation is an integro-differential one even in the case of uniform composition with regard to the radiation process (photosensitized reaction) [3].

While a statistical approach based on Monte Carlo technique appears to be the best tool to handle situations of finite geometrical dimensions [3], analytical or semi-analytical solutions are, on the contrary, feasible for one-dimensional situations, mainly for plane slabs. These last geometries must be considered only as a limit configuration of the reaction vessel. They are nevertheless worth being investigated in order to confirm on a stronger computational basis the physically plausible results obtained following the Monte Carlo approach.

A slab of finite width $s$ has been considered with a parallel plane monochromatic radiation of intensity $I_0$ which impinges normally on the plane $x = 0$, while the face $x = s$ has been assumed to be transparent. Only absorption and scattering have been considered within the medium since emission is usually negligible in photochemical processes: only the case of isotropic scattering has been further examined.

For the situation at hand the local rate of energy absorption is given by

$$\dot{e}'''(x) = \beta I(x) \tag{1}$$

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* Work supported by C.N.R., Rome, Italy.